Archimedes, Euclid And Pythagoras As Ancient Greek Mathematicians

Ancient Greek Mathematicians

“Geometry is knowledge of the eternally existent,” (“Sacred Mathematics”). This quotation by Plato, an Ancient Greek philosopher, demonstrates the importance of geometry to the foundations of the universe. Geometry encompasses every aspect of life including architecture, physics, and biology. Teachers around the globe instruct the basics of geometry to teen-aged students every day, yet these self-evident ideas were not always simple. It took the collaboration of many great minds to formulate the mathematical conclusions so easily comprehensible today. Ancient Greece’s thriving civilization allowed great thinkers such as Thales, Pythagoras, Euclid, and Archimedes to flourish through discovery and innovation. Because of the considerable time period, these mathematicians belong to one of two categories: the early mathematicians (700-400 BCE) and the later mathematicians (300-200 BCE). Thales and Pythagoras are early mathematicians, while Euclid and Archimedes are later mathematicians. Their discoveries provided a better understanding of geometry and developed the principle understandings of the world around us, thus providing invaluable contributions to the field of mathematics, especially in geometry.

Thales: The Father of Greek Mathematics

One of the earliest great Greek mathematicians was Thales. Thales (624-560 BCE) was born in Miletus, but resided in Egypt for a portion of his life. He returned to Miletus later in his life and began to introduce and shape his knowledge of astronomy and mathematics to Greece (Allman 7). As an astronomer, he was infamous for accurately predicting the solar eclipse on May 28, 585 BC. But, evidence points to this prediction being a fluke as astronomy at the time was not advanced enough to make such a prediction (Symonds and Scott 2).

Mathematically, however, his contributions are more reputable. Historians believe that Thales introduced the concept of geometry to Greece (“Thales”). Through his use of logical reasoning and his view of geometrical figures as mere ideas rather than physical representations, Thales drew five conclusions about geometry. In circular geometry, Thales proved the diameter of a circle perfectly bisects the circle, and that an angle inscribed in a semicircle is invariably a right angle, (See Figure 1). In trigonometry (the geometry of triangles) he discovered that an isosceles triangle’s base angles are equal. Today, architects still rely on this principle to ensure that steeples and spires on buildings are level. He also proved that triangles with two congruent angles and one congruent side with each other are, in themselves, congruent, as displayed (See Figure 2). Later, artists used this proof in paintings to ensure symmetry, particularly in modern works. Lastly, he proved that when two straight lines intersect, the opposite angles between the two lines equal each other (Symonds and Scott 1), which is crucial to predicting trajectory in physics (See Figure 3).

His version of geometry was abstract for the time period, as “Thales insisted that geometric statements be established by deductive reasoning rather than by trial and error” (Greenberg 6).

He focused on the relationships of the parts of a figure to determine the properties of the remaining pieces of the figure (Allman 7). Through his discoveries, Thales influenced his successors and aided in their discoveries. But, he also applied them practically to Grecian life. The theorems he formed on congruent triangles and their corresponding parts and angles allowed him to more accurately calculate distances, which ultimately aided in sea navigation (Wilson 80), crucial due to this being their main mode of transportation. Thales' ideas also founded the geometry of lines, "which has ever since been the principal part of geometry," (Allman 15). Through his development of this principal part of geometry and his exposure of these ideas to the Greeks, Thales greatly impacted the overall development of mathematics. Historians acknowledged these contributions by naming Thales as one of the Seven Wise Men of Greece ("Thales").

Pythagoras: The Father of Trigonometry

Living from 569-500 BCE, Pythagoras, too, found an interest in mathematics and astronomy as he studied under one of Thales' pupils, Alzimandar. Through his years of research and study of mathematics, Pythagoras attracted a community of followers in his home of Crotona (Wilson 80). Known as the Pythagoreans, scholars credit them with discovering the sum of the angles of a triangle equalling two right angles, or 180 degrees, and the existence of irrational numbers. Another notable accomplishment is the construction of the five regular solids: the tetrahedron, the hexahedron (cube), the octahedron, the dodecahedron, and the icosahedron (See Figure 4) ("Polyhedron."). Later, scientists found these solids to represent the atomic shapes of compounds. Today, students and educators alike most recognize Pythagoras for the Pythagorean theorem, in which "the square of the hypotenuse of a right angled triangle is equal to the sum of the squares of the other two sides" (Symonds and Scott 3), or $a^2 + b^2 = c^2$. This theorem developed the basic principle of trigonometry, which is the basis of physics.

Eventually, however, the Pythagoreans particularly focused on abstract rather than concrete problems. (Symonds and Scott 3). Rather than focusing on measurable, concrete quantities as numbers, the "Pythagorean worldview was based on the idea that the universe consists of an infinite number of negligibly small indivisible particles" (Naziev 175). This group believed that the objects around them (water, rocks, materials) were all constructed of microscopic, single units, later discovered to be atoms. It is through this assertion that Pythagoras coined his slogan, "All is number." Through this sentiment, he implies that everything in the universe can be explained, organized, and predicted using numbers and mathematics. (M. B. 47).

Euclid: The Father of Geometry

Euclid, the first well-known mathematician from Alexandria, lived from 325-265 BCE. (Wilson 96). Euclid attended a Platonic school, where he found his passion for mathematics and logic (Greenberg 7). He is most well known for his collection of his plane and solid geometry studies: his book Elements. Influenced by Thales' geometrical beliefs, Euclid wrote his Elements to serve as an example of deductive reasoning in practice, starting with "initial axioms and deduc[ing] new propositions in a logical and systematic order." (Wilson 96). Consisting of thirteen books covering topics from arithmetic, plane and solid geometry, and number theory, its groundbreaking content and overall influence catapulted this work to become one of the greatest textbooks in history, being the second most sold book only to the Bible. And, because of the success of this title, experts recorded Euclid as the most widely read author in history

(Greenberg 7), in addition to one of the greatest mathematicians of all time (Symonds and Scott 4).

The first four volumes of Elements focus on the Pythagoreans and some of their discoveries (Greenberg 7). The fifth volume is said to be the “finest discovery of Greek mathematics” as it explains geometry as dependent on recognizing proportions, and the sixth volume applies these proportions to plane geometry. Volumes seven through nine focus on number theory, while volume ten deals with irrational numbers. Lastly, the eleventh through thirteenth volumes focus on three-dimensional geometry (Symonds and Scott 4). Some examples of the content in these volumes include the five postulates in volume one. Euclid writes,

Let the following be postulated: 1. To draw a straight line from any point to any point. 2. To produce a finite straight line continuously in a straight line. 3. To describe a circle with any centre and distance. 4. That all right angles are equal to one another. 5. That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines . . . meet on that side on which are the angles less than the two right angles. (Euclid 2)

Through these postulates, Euclid focuses his propositions, through which he makes discoveries including measurements of angles in constructions, bisections, and proportional lengths (Euclid 2-36).

Euclid’s discoveries span across multiple principles of geometry. These discoveries serve crucial roles in construction and architecture, helping build and produce buildings with structural integrity while aiding the workers in accurately estimating the amount of material needed to complete a project. Not only do these aid in construction, but his discoveries apply greatly to engineering and physics. Through his focus on angles, he created the basis for future scientists to predict trajectory and to aid athletes in optimizing their performance.

Archimedes: The Father of Mathematics

Born in 287 BC, Archimedes of Syracuse on the island Sicily studied mathematics and, because of his discoveries, scholars consider him to be one of the top-ranking mathematicians of all time (Symonds and Scott 4-5). Archimedes continued some of the work of the Pythagoreans and Euclid as he recorded the thirteen semi-regular solids (See Figure 5). Scientists later discovered that these solids serve as depictions of some crystalline structures. Influenced by Euclid’s Elements, he also found the surface areas and volumes of spheres and cylinders used to determine the amount of a substance in a can or the amount of air needed to expand a balloon to a specific size (Wilson 96). Following this discovery, Archimedes detailed his ability to find these properties, saying, “These properties were all along naturally inherent already in their figures referred to, but they were unknown to those who were before our time engaged in the study of geometry, because none of them realized that there exists symmetry between these figures” (Dijksterhuis 142), just as Euclid determined in the fifth volume of Elements. Archimedes found these surface areas and volumes by calculating the ratios for these solids to circles and to each other. For instance, the surface area of a sphere is 4?r2 units while the area of a circle is ?r2 units, leaving the surface area of a sphere and the area of a circle in a perfect 4:1 ratio. Similarly, the volume of a cylinder is ?r2h units while the area of a circle is ?r2 units, meaning these two properties are also directly proportional.

While recognizing the proportionality of these components, Archimedes also decided to focus on “the ratio of the circumference of a circle to its diameter”: pi. By drawing polygons with numerous sides inscribed in a circle and calculating the perimeter of said polygons, he was able to more accurately compute the value of pi. He found pi to lie between 3 10/71 and 3 1/7, the most precise prediction for his time period. (Symonds and Scott 5). This estimation for pi allowed a more accurate calculation of the area of a circle and volumes of spheres and cylinders, which can be applied to the calculation of sound waves to better understand pitch for music.

In computing these components of circular objects, Archimedes actually perfected a method of integration (Symonds and Scott 5), commonplace in calculus which, as a section of mathematics, wasn’t invented until much later. Archimedes was way ahead of his time in discovering this method, and this allowed himself and future mathematicians to calculate areas and volumes for various shapes.

Discussion

The Ancient Greek mathematicians contributed to mathematics more than they could have predicted. Many of these people found interest in the field through their studies of prior mathematicians, and capitalized on prior discoveries to draw their own conclusions. This group of people were some of the first to study principles that were abstract and did not require physical tests to prove; rather, they relied on deductive reasoning to develop their theorems. This practice set the precedent for all future scientific and mathematical discoveries. The Ancient Greek mathematicians influenced not only the mathematics of their times or the mathematics of the future, but the overall process of all further scientific discoveries and experiments, thus proving to be invaluable assets to both the field of mathematics and scientific thought as a whole.