Teaching Algebraic Thinking Through Problem Solving

One organization that provides research-based strategies and best practices for increasing the effectiveness of mathematics educators worldwide is the National Council of Teachers of Mathematics (NCTM). Hiebert found (as cited in Walle, Kark, & Bay-Williams, 2013) that countries whose teaching practices were most similar to those advised by the NCTM scored higher in mathematical achievement. Specifically, Hiebert found that higher scoring countries taught math through problem solving and focused on building conceptual understanding as opposed to teaching procedures for solving specific types of problems (Walle et al., 2013). The NCTM categorizes math into five content standards, which define the limits and expectations for specific content mastery for each grade level, and five process standards, which describe methods for how students learn and apply the concepts from the content standards (NCTM, n.d.-a). This essay will use the framework developed by the NCTM to evaluate how the video of the lesson, “Teaching Math: Staircase Problem”, fulfills the algebra content standard and the process standards for problem solving and representations (Roche, 1996).

The NCTM identifies that the key components of the algebra content standard are to “represent and analyze mathematical situations using algebraic symbols; understand patterns, relations, and functions; use mathematical models to represent and understand quantitative relationships; and to analyze change in various contexts” (NCTM, n.d.-b, para. 1). The staircase problem video addresses all of the components of the algebra content standard for grades 9-12, with the exception of analyzing change, through the context of problem solving. In the video, students were given a visual representation of the pattern used to build a staircase with one, two, or three steps. Students were then asked to work together in small groups to determine how many blocks would be needed to build a staircase with 50 steps, 100 steps, or N steps (Roche, 1996). One reason the staircase problem activity is so effective is because it has multiple access points, which allow students of all levels to approach the problem using different strategies (Walle et al., 2013). Students were given a relatively simple situation and were asked to draw conclusions that required algebraic reasoning (Roche, 1996). The overall design of this activity leads to the obvious conclusion that students are learning algebraic reasoning through the NCTM process standard for problem solving (NCTM, n.d.-a). Students were given a problem, finding how many blocks would be needed for a staircase with N steps, that requires students to use algebraic reasoning. The second process standard used in this video was communication. Jesse Solomon, the teacher in the staircase video, encouraged students to evaluate each other’s ideas by deliberately avoiding giving his own feedback or answering questions (Roche, 1996, 6:25). Instead of correcting students’ mistakes, Solomon asked “why” questions to encourage students to evaluate each other’s reasoning (Roche, 1996, 3:30). At the end of the video, Solomon concludes the lesson by instructing students to write down their reasoning from that day’s activity (Roche, 1996, 14:25). By giving the students time to reflect and write about their reasoning, he is promoting communication about mathematics through writing.

The first component of the NCTM algebra content standard addressed in the staircase problem activity was analyzing patterns and relations. First, students realized that in order to find how many blocks would be needed for a staircase with 50 steps, they would need to identify a pattern to be able to predict how many blocks the next step would have. Students used various strategies with multiple mathematical representations to identify the pattern. Most students

looked for the pattern first by drawing the staircase (or using the square tiles to model the staircase) at each step and then by creating a table to organize the data. Students used a variety of strategies with different degrees of success to determine the pattern and extrapolate a rule about how many blocks would be used for a staircase with N steps. One group skipped drawing the diagram and started with the table using the data from the first three steps given in the instructions (Roche, 1996, 2:45). This group looked at the change in blocks from just the second and third step and determined incorrectly that it was increasing by a constant rate of three blocks for every step. The group also incorrectly applied the rule they determined because they didn't take into account that first step was one block, not three blocks. Unfortunately, just as the third group member begins to disagree, the camera switches over to another group (Roche, 1996, 3:05).

The second NCTM algebra standard this activity addressed was using multiple mathematical models to better understand the problem (NCTM, n.d.-b). Students were encouraged to use drawings, tiles, tables, and eventually develop an equation to represent the staircase problem (Roche, 1996). Solomon noticed that while most groups had a conceptual understanding of how a rule relates to a pattern, students were struggling with the process of how to determine a rule for the specific situation (Roche, 1996, 3:55). Again, Solomon advised students to generate data to look for a pattern first before attempting to find a rule (Roche, 1996). Most groups built or drew staircases and looked at how many blocks were being added each time. However, other groups viewed the staircase as a square of N by N blocks with a triangular section removed (Roche, 1996, 12:20). All of the groups used the drawings, tiles, and tables to help them to create a rule and to challenge their rule's validity.

Understanding patterns, relations, and functions was the final NCTM algebra content standard addressed in the video (NCTM, n.d.-b). At least one group was able to determine the pattern and created a recursive formula that identified that the total number of blocks equals the total number of blocks from the previous step plus the number of current steps (Roche, 1996, 10:55). After groups discovered the recursive function for the staircase problem, the teacher asked them to determine if the relation could be written without needing to know the number of blocks in the previous step (Roche, 1996, 11:50). At the start of the video, most groups struggled to see find the easier recursively defined rule because they were expecting a one step or two step explicitly defined rule. Once students were able to stop worrying about what the rule would look like, and instead began to focus on the data, they were able to see, and eventually articulate, the pattern. This learning activity encouraged many groups to reevaluate how they analyze patterns to determine a function.

The staircase learning activity promoted algebraic thinking through the context of the two process standards for problem solving and communication. This problem-based lesson encouraged students to represent situations using symbols, use multiple models to represent relationships, and to understand how to create recursively and explicitly defined functions from a pattern (NCTM, n.d.-b, para. 1). This lesson was an excellent example of how problem-based learning that is aligned to content standards can engage students in productive struggle and lead to the development of a deeper conceptual understanding of algebraic reasoning.

References

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